Math330 HW1 (Fall 2020)

Professor Youngjoon Hong — Due Date: Sep. 4 (11:59 pm)

Problem 1 *Give the contraposition of the following statements:*

$$(a) \quad x^n > 1 \implies f(x) < 2 :$$
$$(b) \quad 3x + 1 = 0 \implies x = 13 :$$

Problem 2 *Give the negation of the following statements:*

$$(a) \quad \forall \epsilon > 0, \exists N \in \mathbb{N}, \forall n \in \mathbb{N}, |Na_n - 1| < \epsilon$$
$$(b) \quad \forall \epsilon > 0, \exists \delta > 0, \forall x \in \mathbb{R}, |x - l| < \delta \implies |f(l) - L| < \epsilon :$$

Problem 3 *Use the mathematical induction to prove that for a natural number n,*

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$

Problem 4 *Use the mathematical induction to prove that for $x > -1$,*

$$(1+x)^n \geq 1 + nx, \quad \forall n \in \mathbb{N}.$$

Problem 5 *Prove that for any numbers a and b,*

$$ab \leq \frac{1}{2}(a^2 + b^2).$$