Problem 1 *Prove that*

$$\frac{d}{dx}\sin(x) = \cos(x), \quad \frac{d}{dx}\cos(x) = -\sin(x).$$

Hint: use the fact $\lim_{u \to 0} \frac{\sin u}{u} = 1$.

Problem 2 *Use the formula of the derivative of an inverse function to prove that* $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Problem 3 *Let*

$$f(x) = \begin{cases} 2x, & \text{if} \quad 0 \leq x \leq 1, \\ -x+3 & \text{if} \quad 1 < x \leq 3. \end{cases}$$

Determine if Rolle's theorem is verified on $[0,3]$.

Problem 4 *Find the values of* c *such that the Mean-Value Theorem is satisfied for* $f(x) = -2x^3 + 6x - 2$ *on* $[-2,2]$.

Problem 5 *Use the Mean-Value Theorem to prove $\forall a, b \in \mathbb{R}$, $|\cos a - \cos b| \leq |a - b|$.*