

Math330 HW2 (Fall 2020)

Professor Youngjoon Hong

Due Date: Sep. 11 (11:59 am)

Problem 1 *Show that the triangular inequality becomes an equality if a and b are of the same sign.*

Problem 2 *In class, we proved the following argument: If x and y are any real numbers with $x < y$, then there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$. Using this density argument, show that there exists an irrational number z such that $x < z < y$ for any real number x and y .*

(Hint: As $\sqrt{2}$ is an irrational number, we consider the interval $(x/\sqrt{2}, y/\sqrt{2})$. Then apply the density argument of \mathbb{Q} .)

Problem 3 Use the ϵ notation to prove the following limits (I expect all details here).

$$(a). \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0.$$

$$(b). \lim_{n \rightarrow \infty} \frac{n^2}{n^2 + n} = 1.$$

$$(c). \lim_{n \rightarrow \infty} b^n = 0, \quad 0 < b < 1.$$