Math330 HW3 (Fall 2020)

Professor Youngjoon Hong — Due Date: Sep. 23 (11:59 pm)

Problem 1 *Let $\{a_n\}$ a sequence converging to $L > 0$. We also assume that $\forall n \in \mathbb{N}$, $a_n \geq 0$. Show that the sequence $\sqrt{a_n}$ converges to $\sqrt{L}$.*
Hint: if $L > 0$, then $\sqrt{a_n} - \sqrt{L} = \frac{a_n - L}{\sqrt{a_N} + \sqrt{L}}$.

Problem 2 *Let $\{a_n\}$ a sequence converging to $L > 0$. Show that $\exists N \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq N$, $a_n > 0$.*

Problem 3 *Let* $\{a_n\}$ *a sequence converging to* L*. Let* $\{b_n\}$ *a sequence such that* $\exists N_b \in \mathbb{R}, \forall n \in \mathbb{N}, n \geq N_b,\ a_n = b_n$*. Show that* $\{b_n\}$ *converges to* L *as well.*

Problem 4 *The following questions will guide you to prove that* $\lim_{n\to\infty} n^{1/n} = 1$*.*
1. Denoting $\alpha_n = n^{1/n} - 1$*, explain why*

$$\lim_{n\to\infty} n^{1/n} = 1 \iff \lim_{n\to\infty} \alpha_n = 0.$$

2. Using the Binomial formula, show that $\forall n \in \mathbb{N}$*,*

$$n = (1+\alpha_n)^n \geq 1 + \frac{n(n-1)}{2}\alpha_n^2.$$

3. Show that $\lim_{n\to\infty} \alpha_n = 0$ *and conclude that* $\lim_{n\to\infty} n^{1/n} = 1$*.*