Math330 HW4 (Fall 2020)

Professor Youngjoon Hong | Due Date: Sep. 30 (11:59 pm)

Problem 1 *Use the formal definition of a Cauchy sequence to show that the sequence*

$$\{a_n\} = \left\{ \frac{n+5}{6n+1} \right\}$$

is a Cauchy sequence.

Problem 2 *Let a sequence* $\{a_n\}$ *satisfying the condition*

$$\forall c \in (0,1), \forall n \in \mathbb{N}, |a_{n+2} - a_{n+1}| < c|a_{n+1} - a_n|.$$

1. *Show that*
$$\forall c \in (0,1), n \geq 2, |a_{n+1} - a_n| < c^{n-1}|a_2 - a_1|.$$
2. *Show that* a_n *is a Cauchy sequence.*

Problem 3 *Let S be the set of points defined by*

$$S = \left\{ n \in \mathbb{N} : 1 - \frac{(-1)^n}{n} \right\}.$$

Show that $\sup S = 2$ *and* $\inf S = 1/2$.

Problem 4 *Let S be a nonempty subset of $\mathbb{R}$ that is bounded above, and let a be any number in $\mathbb{R}$. Define the set $a + S := \{a + s : s \in S\}$. Prove that*

$$\sup(a + S) = a + \sup S.$$