

## Math336 HW1 (Spring 2020)

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Due Date: Feb. 11 (3:30 pm)

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**Problem 1** *Your grandparents have an annuity. The value of the annuity increases each month as 1% interest on the previous month's balance is deposited. Your grandparents withdraw \$1000 each month for living expenses. Currently, they have \$50,000 in the annuity.*

(a). *Model the annuity with a dynamical system.*

(b). *Find the equilibrium value.*

(c). *Build a numerical solution of the dynamical system using the initial condition and find the explicit solution after 2 years (24 months). You do not need to evaluate exponents.*

**Problem 2** *It is observe that a high price for the product in the market attracts more supplier. Increasing the quantity of the product supplied tends to drive the price down. Over time, there is an interaction between price and supply. The economist has proposed the following model:*

$$\begin{aligned}P_{n+1} &= P_n - 0.1(Q_n - 500), \\Q_{n+1} &= Q_n + 0.2(P_n - 100)\end{aligned}$$

where  $P_n$  represents the price of the product at year  $n$ , and  $Q_n$  the quantity.

- (a). Find an equilibrium value if exists.
- (b). Using the following initial conditions  $(P_0, Q_0)$ ,

$$(100, 500), \quad (200, 500), \quad (100, 600), \quad (100, 400),$$

predict the long-term behavior as in the lecture.

**Problem 3** *An object is sliding down a ramp inclined at an angle of  $\theta$  radians and attains a terminal velocity before reaching the bottom. Assume that the drag force caused by the air is proportional to  $Sv^2$ , where  $S$  is the cross-sectional area perpendicular to the direction of motion and  $v$  is the speed. Further assume that the sliding friction between the object and the ramp is proportional to the normal weight of the object. Determine the relationship between the terminal velocity and the mass of the object. If two different boxes, weighing 600 and 800 lb, are pushed down the ramp, find the relationship between their terminal velocities.*

**Problem 4** *Find a quick tutorial on MATLAB. Read them and practice the examples.*