

Problem 1 *Derive the weak form of the BVP:*

$$\begin{aligned} -\nabla \cdot (\kappa \nabla u) &= f, & \text{in } \Omega, \\ u &= g, & \text{on } \Gamma_2 \\ \kappa \frac{\partial u}{\partial n} &= h, & \text{on } \Gamma_1. \end{aligned}$$

Problem 2 Consider the 1D BVP

$$\begin{aligned} -u'' &= f \quad \text{in } (0, 1), \\ u(0) &= 0, \\ u(1) &= 0, \end{aligned}$$

where $f(x) = e^x$. Define W to be the subspace of $H_0^1(0, 1)$ spanned by the basis

$$\{x(x-1), x(x-1/2)(x-1), x(x-1/3)(x-2/3)(x-1), x(x-1/4)(x-1/2)(x-3/4)(x-1)\}.$$

Apply the Galerkin method to find the best approximation in the energy norm from W to the solution u .

Problem 3 (*Programming exercise*). Consider the 1D BVP

$$\begin{aligned} -u'' &= f \quad \text{in } (0, 1), \\ u(0) &= 0, \\ u(1) &= 0, \end{aligned}$$

where $f(x) = e^x$. Use the finite element method to find the numerical solution of u .